Normalisation in Coulex experiments

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- Normalisation constants in GOSIA:
  - independent normalisation
  - user-given normalisation constants
- Possible methods of normalisation:
  - elastic scattering, known lifetimes, target excitation...

Big thanks to Kasia Wrzosek-Lipska for many of the slides!

Normalisation in Coulex experiments

- in Coulex experiments, we measure gamma-ray intensities as a function of particle scattering angle
- we need to convert those to absolute excitation cross sections
- without a proper normalisation, a GOSIA fit can converge, but the results will be wrong!
- to relate experimentally measured and calculated gamma-ray intensities, normalisation constants *C* are introduced in GOSIA:

$$\chi^2 = \sum_m \sum_i (C_{\text{global}} C_m I_i^c - I_i^e)^2/\sigma_i^2 + \ldots$$

  - I_i^e: experimental γ-ray intensity for the i-th transition
  - I_i^c: γ-ray intensity calculated by GOSIA for the i-th transition
  - σ_i: experimental uncertainty of the i-th gamma-ray intensity
  - C_m: normalisation constant for the m-th experiment – may be specified by the user of fitted in the minimisation process
  - C_{global}: global normalisation constant – always fitted in the minimisation process

Normalisation constants in GOSIA

- What is their physical meaning?

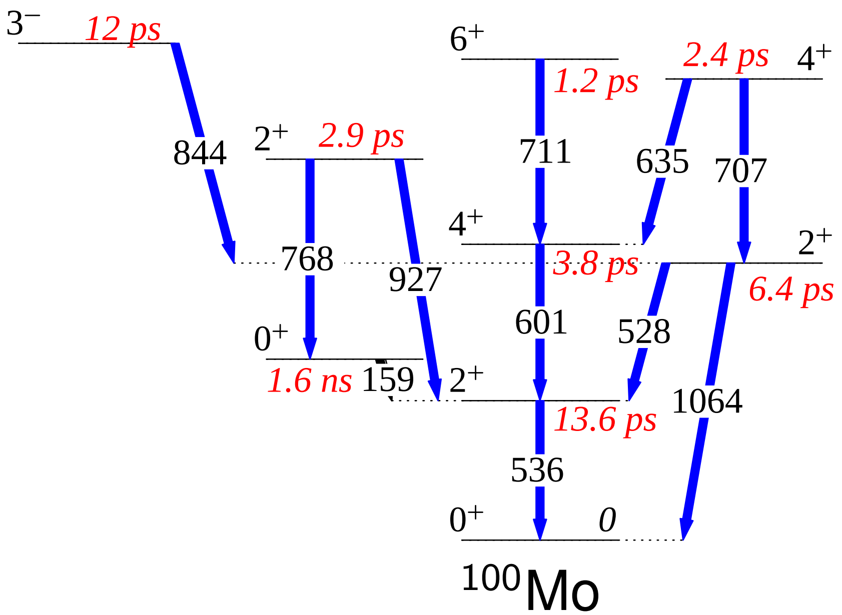
$$\chi^2 = \sum_m \sum_i (C_{\text{global}} C_m I_i^c - I_i^e)^2/\sigma_i^2 + ...$$

- product of:
  - Rutherford cross section
  - absolute efficiency of particle and gamma detection (including solid angle covered by the particle detectors)
  - time-integrated beam current
- relative normalisation constants C_m can be either specified by the user:
  - they can be determined independently from the number of quasi-elastically scattered particles, or from target excitation *(will be discussed later)*
- or fitted in the minimisation process:
  - GOSIA calculates for each data set normalisation constants C_m that yield a minimum χ^2 value for a given set of ME's

Independent normalisation of data sets in GOSIA

- when should it be used?
- always if possible!

- number of experimental data points considerably larger than the number of ME's to be fitted – no problem to introduce few more parameters in the GOSIA fit



K. Wrzosek-Lipska et al., PRC 86, 064305 (2012)

- multi-step Coulex (with stable beams):
  - lifetimes (+ other spectroscopic data) known – usually overdetermined cases
  - beam intensities $\sim 10^9$pps – high statistics

Normalisation to known transition probabilities

- since the global normalisation constant C_{global} is always fitted by GOSIA during the minimisation procedure, it is possible to rescale matrix elements to obtain the same $C_{\mathrm{global}} C_m I_i^c$
  - always true for excitation directly from the ground state
  - more complicated dependence for multi-step excitation *(will be discussed later)*
- an independent measurement yielding a value of a $\langle g.s.\|E2\|I_f\rangle$ is enough to convert all measured gamma-ray intensities to absolute cross sections
- usually fulfilled for stable and neutron-deficient nuclei:
  - ^{74,76}Kr, E. Clément et al., PRC 75, 054313 (2007)
  - $^{182-188}$Hg, N. Bree et al., PRL 112, 162701 (2014)

Normalisation to known transition probabilities – complications

- for odd-mass or odd-odd nuclei multipole mixing ratios become important (for a mixed transition, a lifetime alone is not enough to determine both matrix elements)

- low-energy transitions in heavy nuclei can also be strongly converted and difficult to be measured in gamma-ray spectroscopy
  - normalisation to the next higher-lying transition usually possible (^{224}Ra, L. Gaffney et al., Nature 497, 199 (2013))

What to do if we don't know any lifetimes in the nucleus under study?

- Common problem for neutron-rich nuclei

- Normalisation to quasi-elastic (Rutherford) cross section – most direct solution, used widely in early days of Coulex
  - precise knowledge of the scattering angular range, absolute efficiency, dead time, beam current is required

Normalisation to target excitation

- number of gamma rays emitted from the target N_t:

$$N_t = L \cdot \frac{\rho d N_A}{A_t} \cdot b_t \epsilon_\gamma(E_t) \epsilon_{\mathrm{part}} \sigma_t$$

  - L: time-integrated luminosity of the beam
  - b_t: gamma-ray ranching ratio for the transition
  - σ_t integrated cross section to excite a given state in the target nucleus

- similarly for the beam:

$$N_p = L \cdot \frac{\rho d N_A}{A_p} \cdot b_p \epsilon_\gamma(E_p) \epsilon_{\mathrm{part}} \sigma_p$$

- if we take the ratio of N_p/N_t all tricky parts cancel out:

$$\frac{N_p}{N_t} = \frac{b_p \epsilon_\gamma(E_p) \sigma_p}{b_t \epsilon_\gamma(E_t) \sigma_t}$$

- σ_t can be accurately calculated if B(E2)'s and Q_s are precisely known $\rightarrow$ we determine σ_b and subsequently ME's in the projectile nucleus

Normalisation to target excitation: beam purity

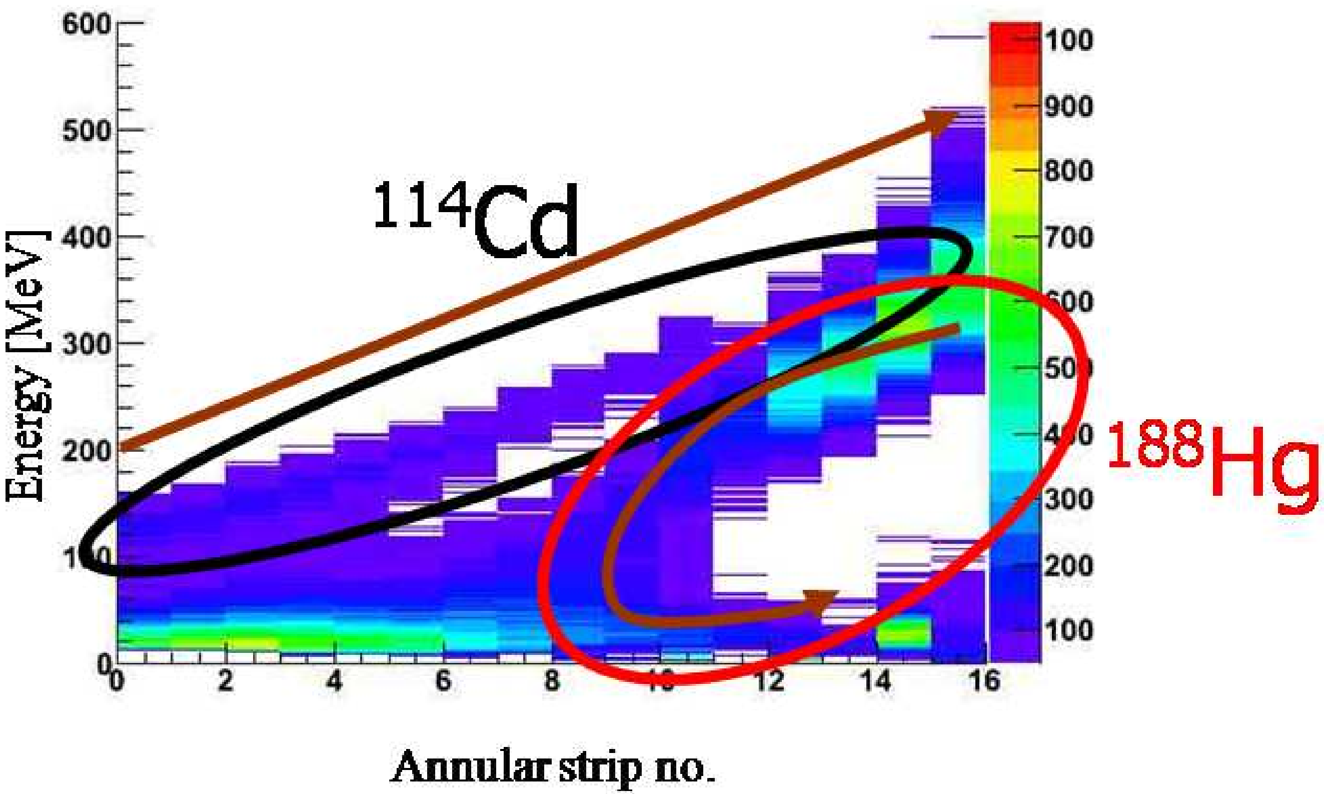
- radioactive beams are not always pure and part of observed excitation of the target is caused by beam impurities
- measured gamma-ray yields for the target should be multiplied by a factor:

$$F = \frac{1}{1 + \sum_c \left(r_c \frac{\sigma_t(Z_c, A_c)}{\sigma_t(Z_X, A_X)} \right)}$$

  - $r_c = I_c / I_X$ (I_X intensity of the beam of interest, I_c of the contaminant)
  - $\sigma_t(Z_c, A_c)$, $\sigma_t(Z_X, A_X)$ cross sections to excite the target
  - usually isobaric contaminants A_c = A_X; but if masses are different, energies are probably different too!

How to choose a good target for a Coulex experiment?

- highest possible Z to enhance Coulex cross section
- unambigous identification of collision partners necessary
- no gamma transitions of similar energy in target and projectile
- electromagnetic structure (B(E2)'s, Q_s well known)

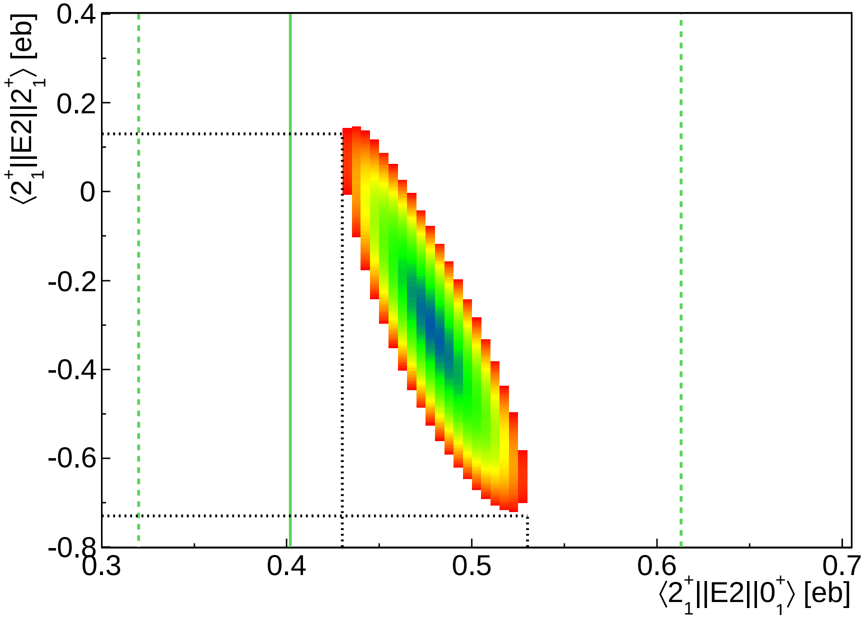


Normalisation to target excitation: GOSIA2

- developed to handle simultaneous analysis of both target and projectile excitation
- limited to one combination of beam and target
- two input files have to be prepared: one for target, one for beam
- GOSIA2 minimises the χ^2 function for the target (this includes calculation of *global* normalisation factors) and then uses the same normalisation factors as a starting point when minimising χ^2 for the beam
- normalisation factors are shared as parameters across both χ^2 functions
- after several iterations the best set of normalisation factors found

Limitations of GOSIA2

- impossible to combine data collected on different targets
- error calculation not incorporated – must be done ”by hand”
- if one-step excitation for both target and projectile, one can use standard error progression (contributions from uncertainties of gamma-ray yields (target and projectile) and of B(E2) in the projectile)
- if two ME's important for the projectile ($\langle 0^+\|E2\|2^+\rangle$ and $\langle 2^+\|E2\|2^+\rangle$) – analysis of χ^2 surface (requesting $\chi^2 < \chi^2_{\rm min} + 1$)



^{96}Sr on ^{109}Ag, ISOLDE

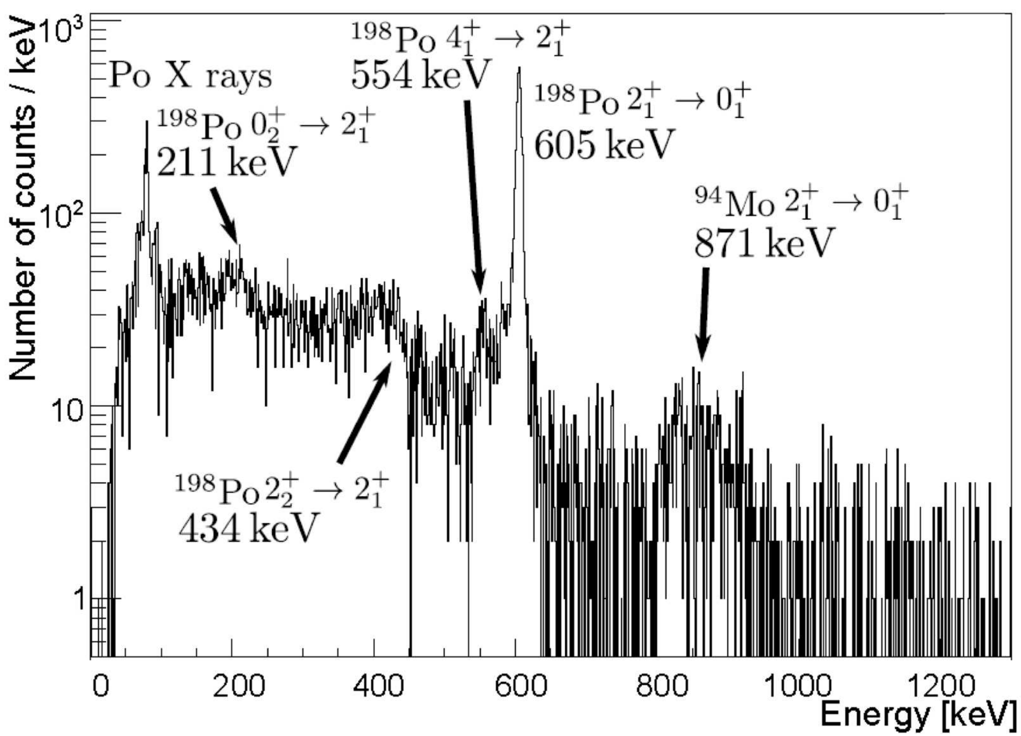
E. Clément *et al.* PRL 116, 022701 (2016)

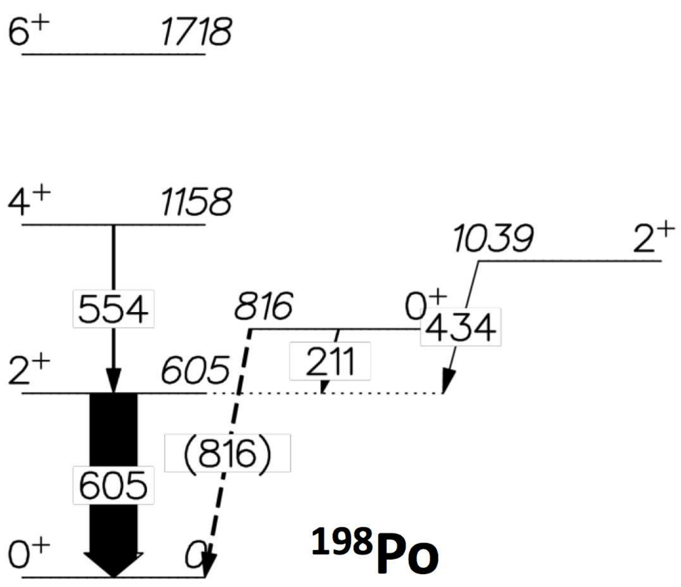
E. Clément *et al.* PRC, accepted

- if more than two matrix elements involved – almost impossible!

Limitations of GOSIA2

N. Kesteloot et al, PRC 92, 054301 (2015)





^{198}Po on ^{94}Mo, ISOLDE

- no complementary spectroscopic data
- more than two matrix elements involved
- normalisation to the target excitation

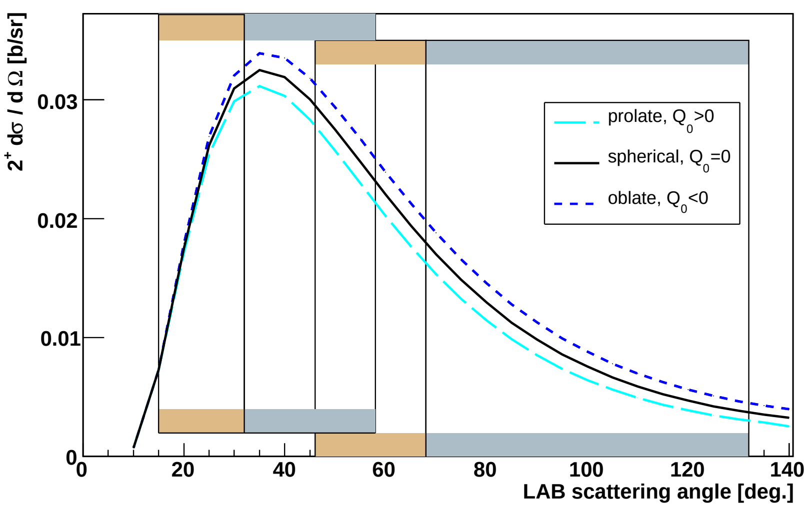
Problems:

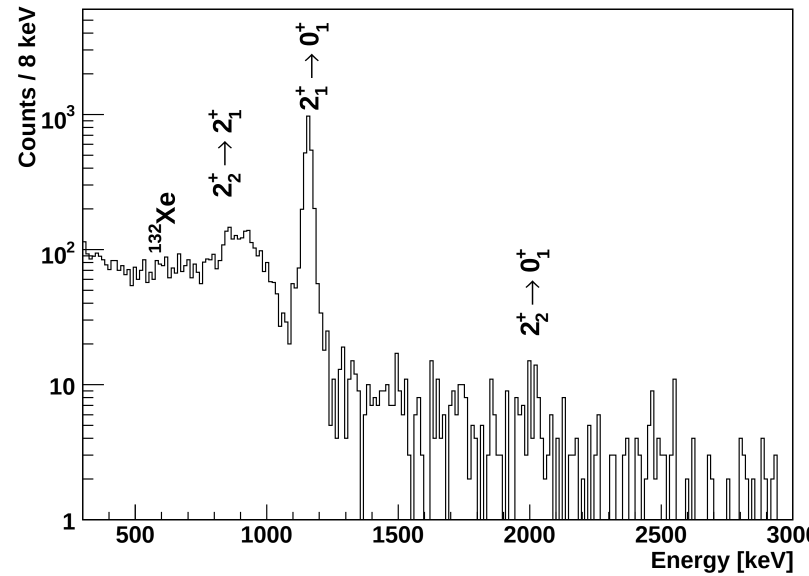
- error calculation!
- how to take into account uncertainty related to target excitation?

Solution 1 (not always possible)

M. Zielińska et al., PRC 80, 014317 (2009)

Example of ^{44}Ar, data collected on ^{109}Ag and ^{208}Pt targets





- lowest angular range for ^{109}Ag target – influence of quadrupole moment negligible $\rightarrow$ determination of B(E2;$2_1^+ \rightarrow 0^+$) (GOSIA2)
- information from other data sets (^{109}Ag and ^{208}Pt targets) $\rightarrow$ determination of quadrupole moment of the 2_1^+ state and other B(E2)'s (standard GOSIA)
- relative normalization of data sets (C_m constants) based on target excitation

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M. Zielińska et al., PRC 80, 014317 (2009)

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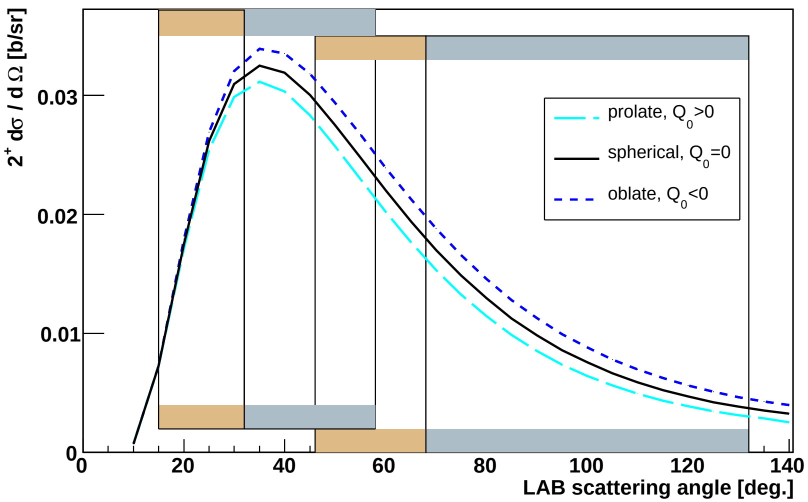
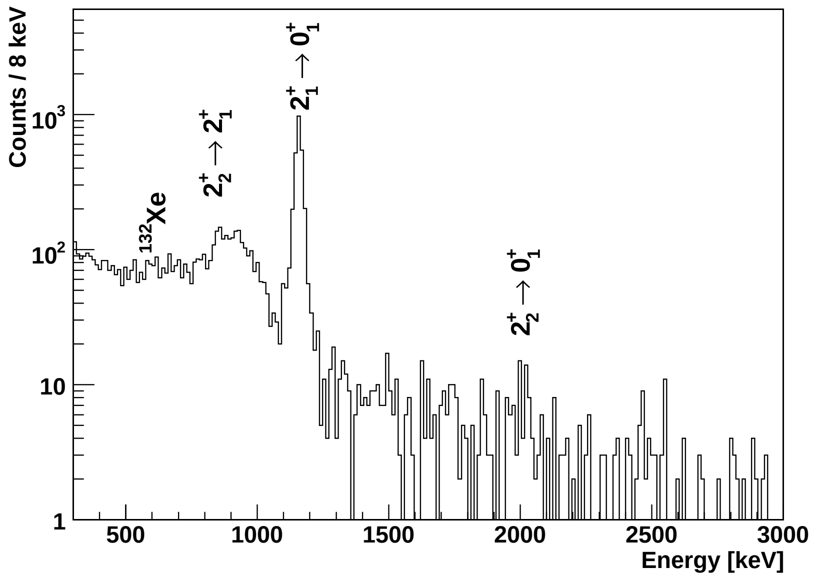
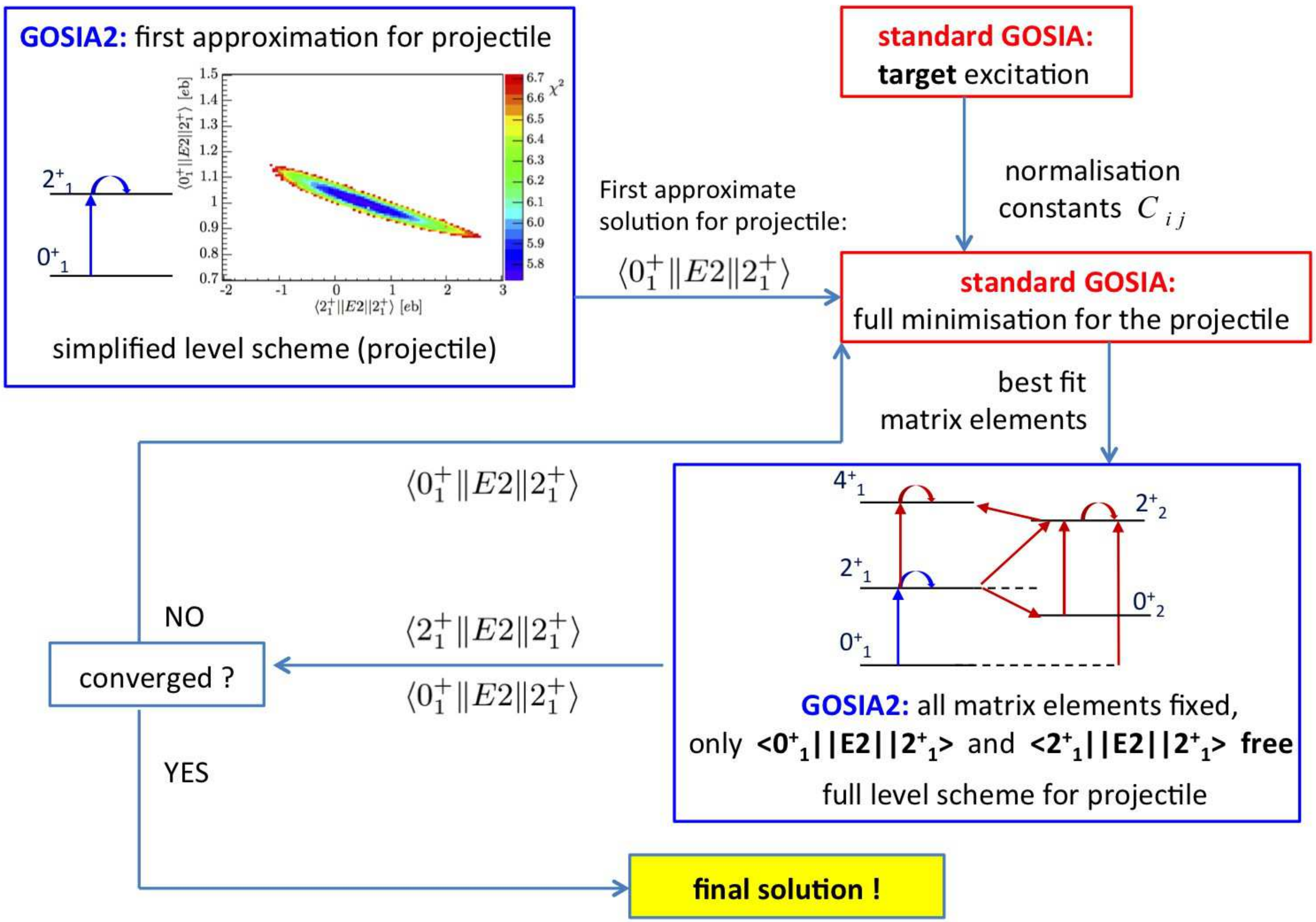




- several methods of data subdivision tested (3, 4, 6, 7, 8 bins)
- compromise between level of statistics in an individual experiment and number of γ-ray yields corresponding to different ranges of the scattering angle
- obtained values of the quadrupole moment consistent
- precision varied from 35 % to 70 %

Solution 2: combined GOSIA - GOSIA2 analysis

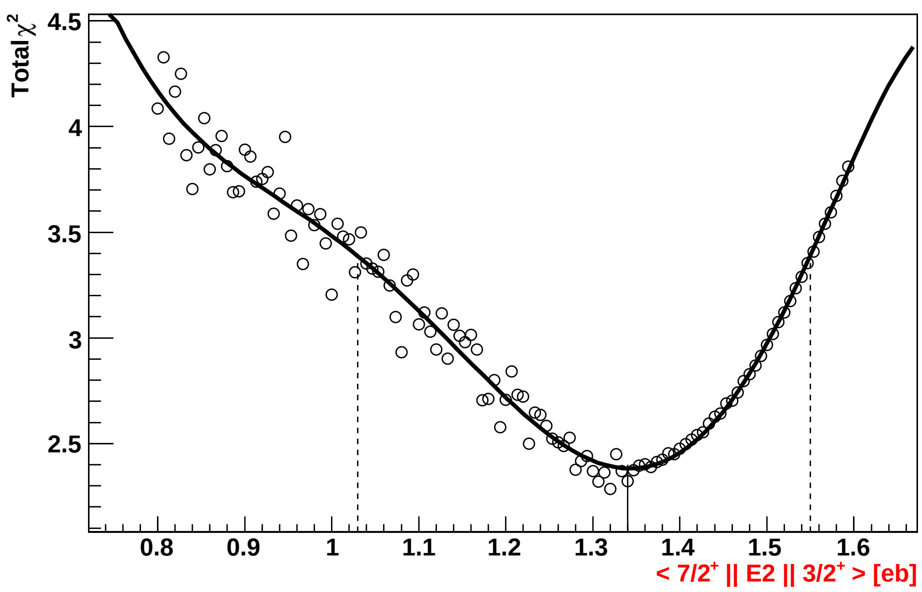


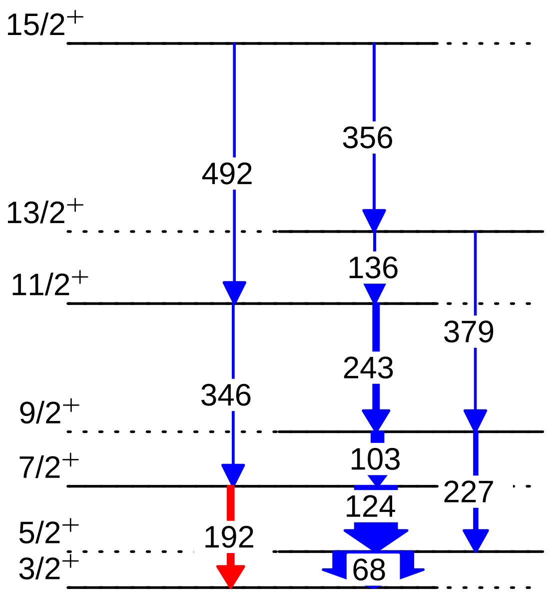
Example of ^{97}Rb: normalisation to target excitation

- for each value of $\langle 7/2^+ \| E2 \| 3/2^+ \rangle$ all remaining matrix elements in Rb and Ni are fitted to observed gamma-ray intensities and known spectroscopic data (GOSIA2)

- Alaga rules assumed for each pair of I $\rightarrow$ I-1 and I $\rightarrow$ I-2 E2 transitions

$$\langle KI_f \| E2 \| KI_i \rangle = \sqrt{(2I_i + 1)} (I_i, K, 2, 0 | I_f, K) \sqrt{\frac{5}{16\pi}} eQ_0$$

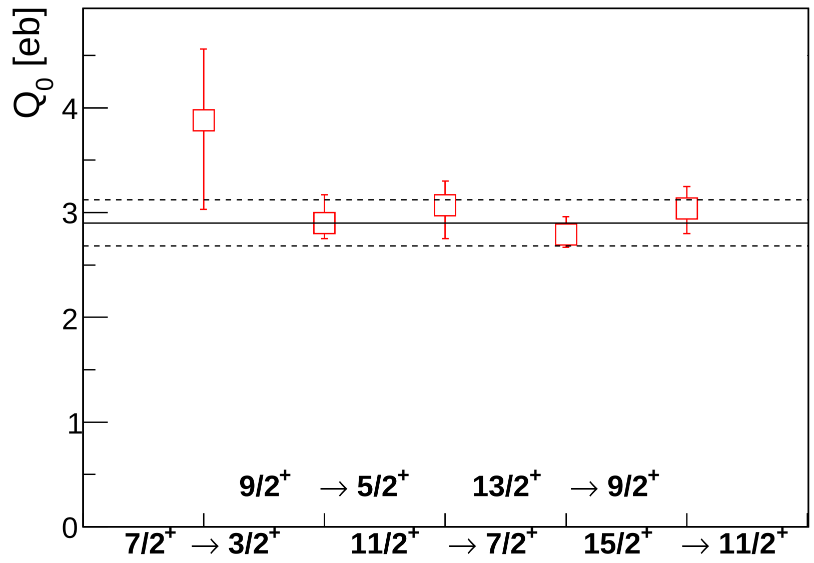


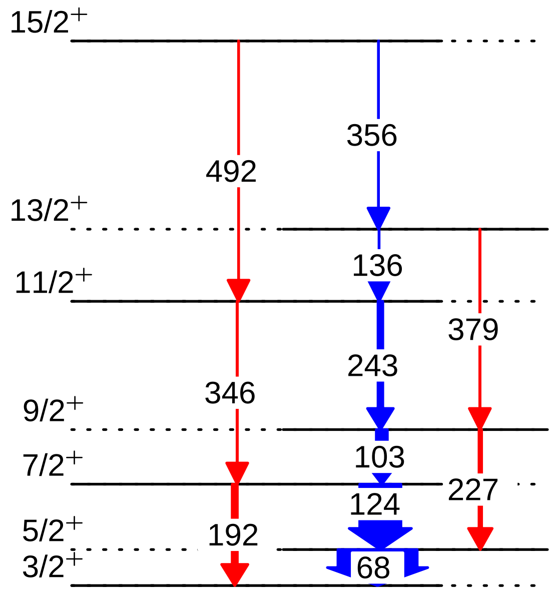


- for all other transitions a standard GOSIA1 analysis assuming this value of $\langle 7/2^+ \| E2 \| 3/2^+ \rangle$

C. Sotty *et al.* Phys. Rev. Lett. 115, 172501 (2015)

Results: deformation of ^{97}Rb





- Alaga rules assumed for each pair of I → I-2 / I → I-1 transitions:

E2 part of a mixed E2/M1 transition determined from the I → I-2 intensity, the remaining part of I → I-1 attributed to M1 decay

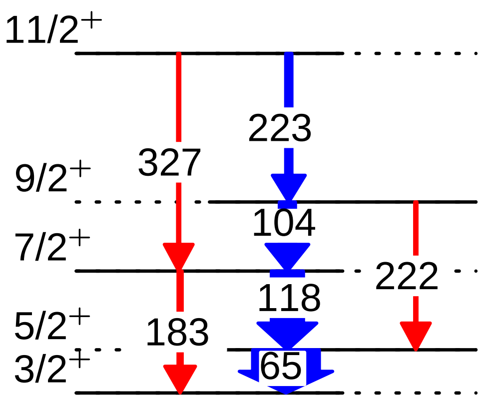
- constant Q_0 within the band

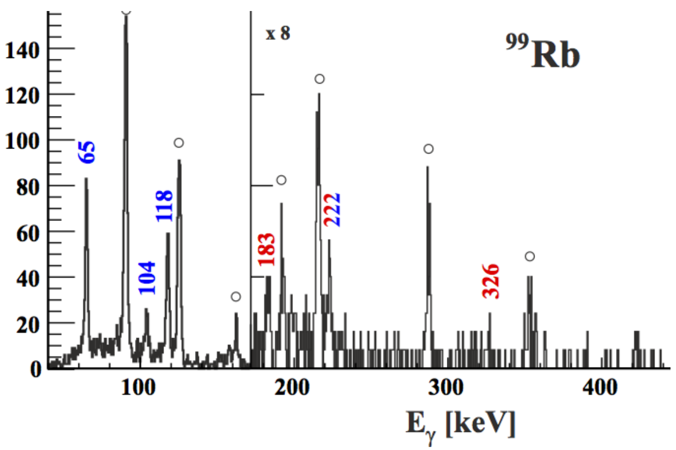
- results consistent with Q_{sp} of the ground state measured in laser spectroscopy

C. Sotty *et al.* Phys. Rev. Lett. 115, 172501 (2015)

Next step: ^{99}Rb

- strong correlations of all matrix elements like in the ^{97}Rb case and...
  - very low statistics (few hundred counts in the strongest line)
  - target excitation not observed
  - unresolved doublet at 222 keV
  - extremely underdetermined problem: 6 gamma rays, 15 matrix elements

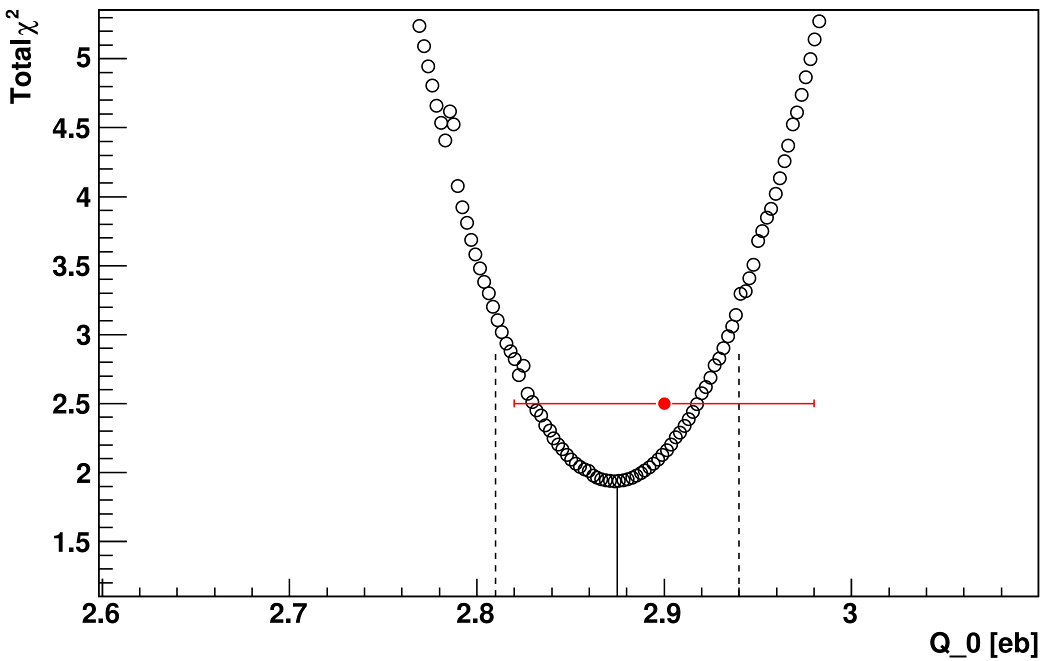


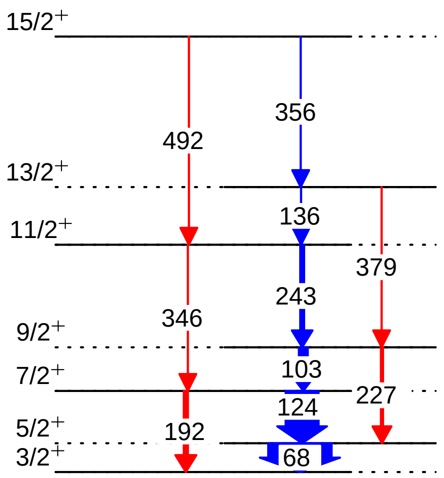


... but matrix elements in the upper part of a strongly deformed rotational band are related to observed intensity ratios in the nucleus under study (no external normalisation required)

^{99}Rb: proposed solution and test on ^{97}Rb data

- all E2 matrix elements (including Q_s) coupled using rotational model
- then we fit only M1 matrix elements and one Q_0 to measured gamma-ray intensities



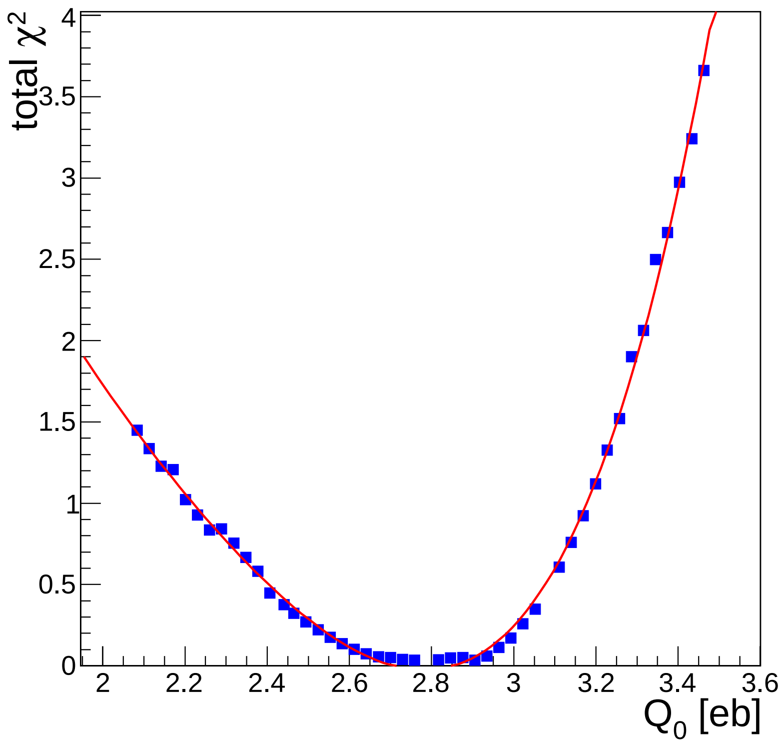


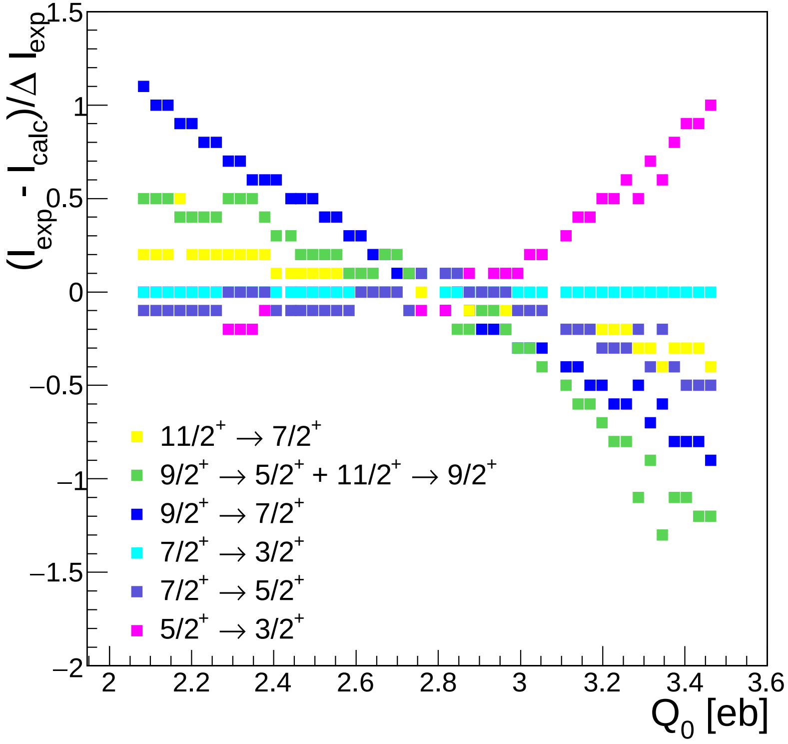
- tested on ^{97}Rb data, result consistent with weighted average of Q_0 values obtained in standard analysis

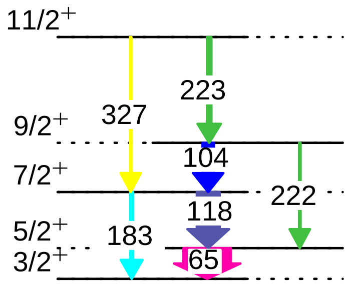
M. Zielińska *et al.* EPJA 52, 99 (2016)

^{99}Rb: results

- 4 M1 matrix elements and one Q_0 fitted to measured gamma-ray intensities in ^{99}Rb



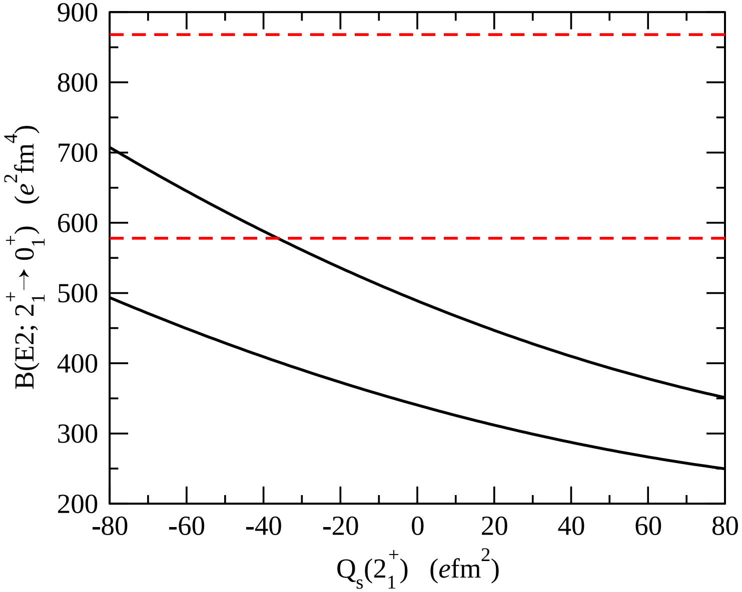




- one clear χ^2 minimum for all observed transitions
- precision rather low due to limited statistics

Additional measurements needed for Coulex data analysis...

- lifetime measurements
  - necessary for integral cross-section measurements



A.M. Hurst *et al.*,

Phys. Rev. Lett. 98, 072501 (2007)

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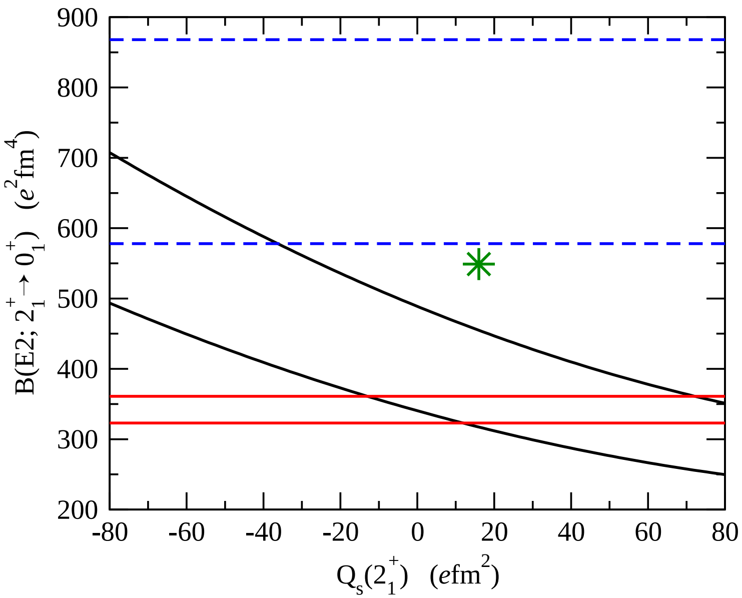

J. Ljungvall *et al.*,
Phys. Rev. Lett. 100, 102502 (2008)

  - increase precision of quadrupole moments/intra-band matrix elements for differential measurements

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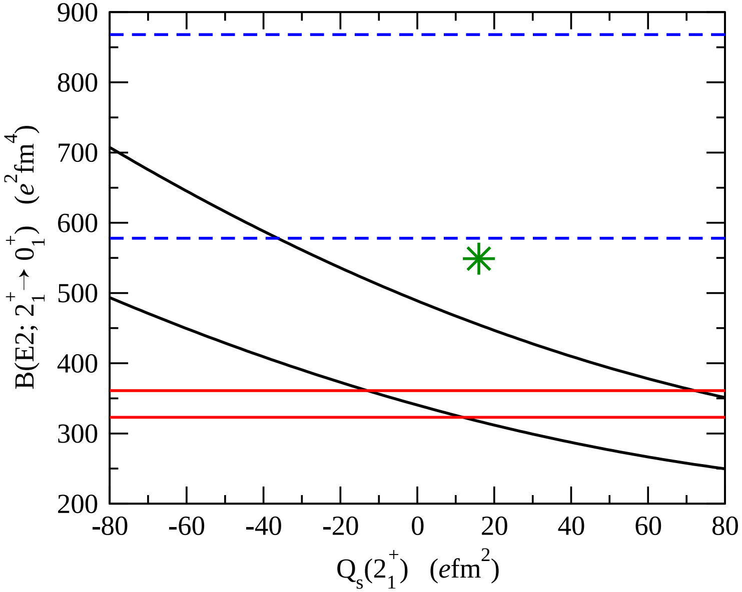
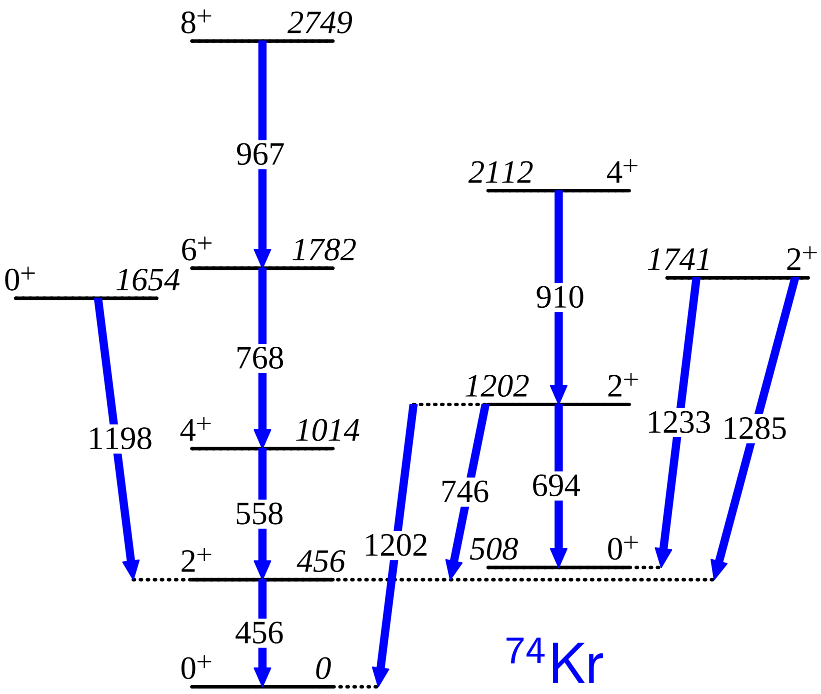


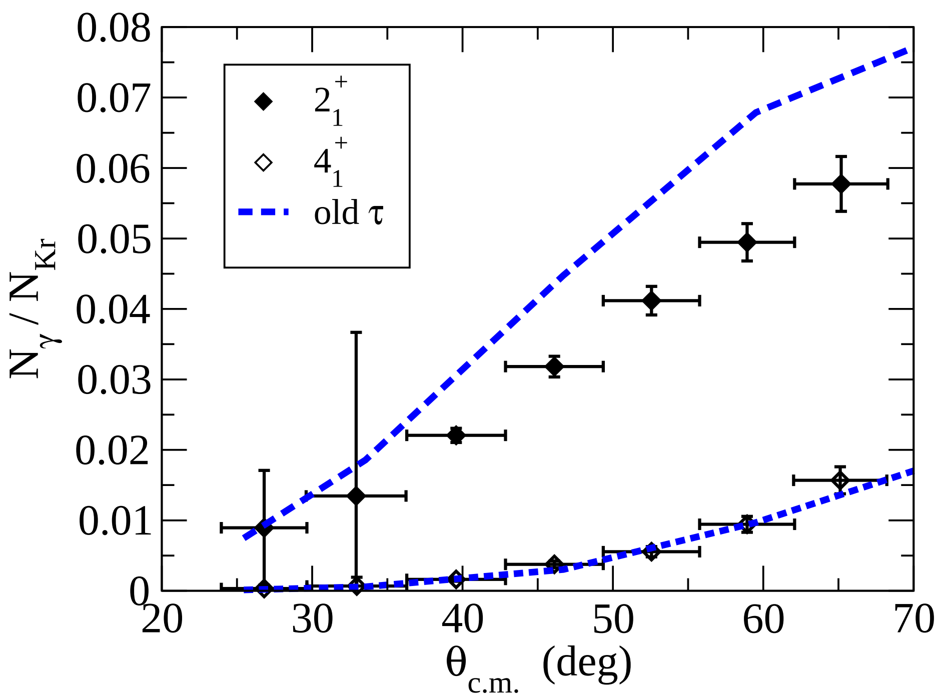
J. Ljungvall *et al.*,
Phys. Rev. Lett. 100, 102502 (2008)

  - increase precision of quadrupole moments/intra-band matrix elements for differential measurements
- beam composition (isobaric contamination/isomeric ratio)
- beam energy
- conversion coefficients/E0 branchings

Coulomb excitation and lifetime measurements



- subdivision of data in several ranges of scattering angle
- spectroscopic data (lifetimes, branching and mixing ratios)
- least squares fit of $\sim$30 matrix elements (transitional and diagonal)



- results inconsistent with previously published lifetimes
- new RDM lifetime measurement:
  Köln Plunger & GASP
  ^{40}Ca (^{40}Ca,α2p) ^{74}Kr
  ^{40}Ca (^{40}Ca,4p) ^{76}Kr

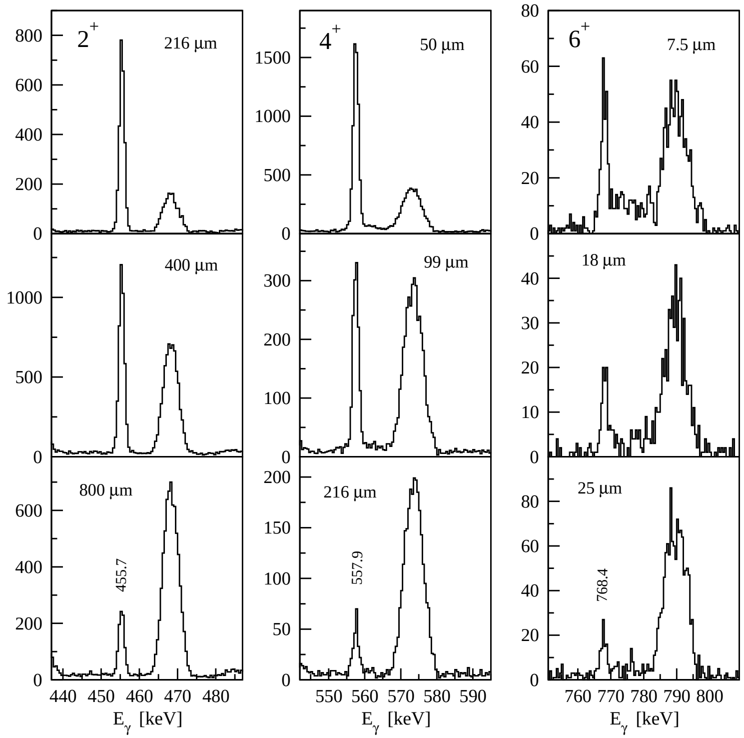
Lifetime measurement

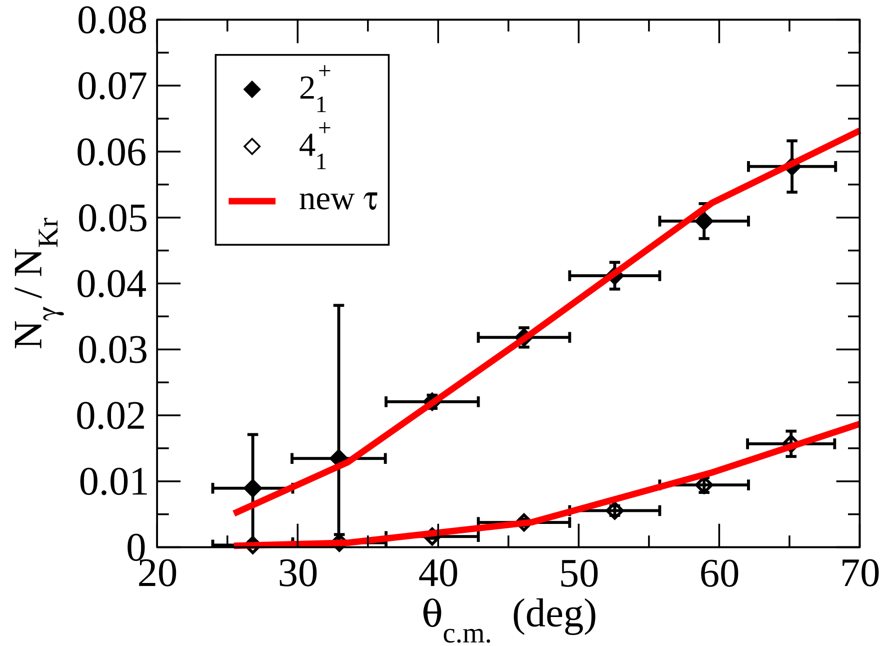
A. Görgen *et al.* EPJ A 26 153 (2005)

		old	new			old	new
^{76}Kr	2^+	35.3(10) ps	41.5(8) ps	^{74}Kr	2^+	28.8(57) ps	33.8(6) ps
	4^+	4.8(5) ps	3.87(9) ps		4^+	13.2(7) ps	5.2(2) ps

^{74}Kr, forward detectors (36°)

gated from above





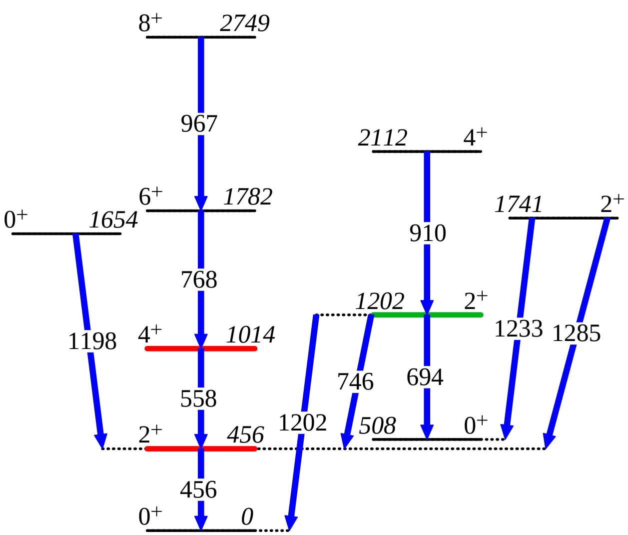
- new lifetimes in agreement with Coulex
- enhanced sensitivity for diagonal and intra-band transitional matrix elements

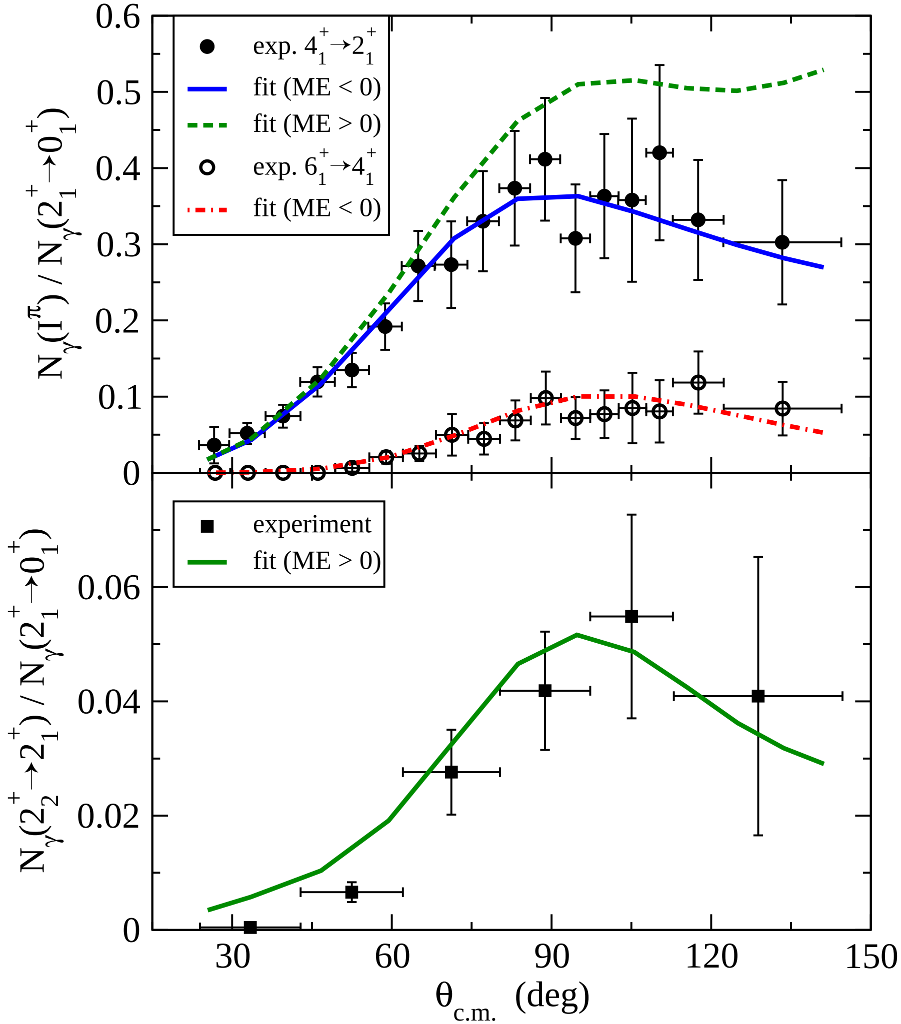
Results: shape coexistence in light Kr isotopes

^{76}Kr: 18 transitional + 5 diagonal ME
^{74}Kr: 14 transitional + 5 diagonal ME

$$\langle 2_1^+ \| E2 \| 2_1^+ \rangle = -\,0.70^{-0.33}_{-0.30}$$
$$\langle 4_1^+ \| E2 \| 4_1^+ \rangle = -\,1.02^{+0.59}_{-0.21}$$

$$\langle 2_2^+ \| E2 \| 2_2^+ \rangle = +\,0.33^{+0.28}_{-0.23}$$



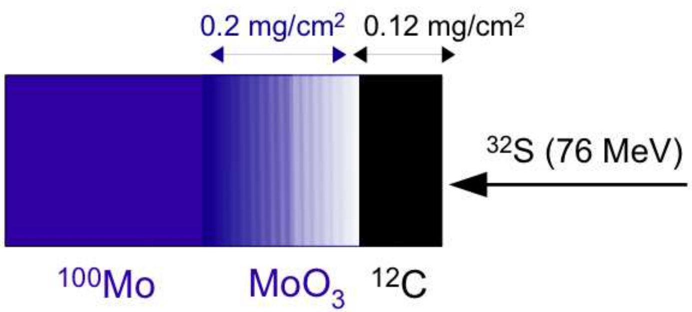


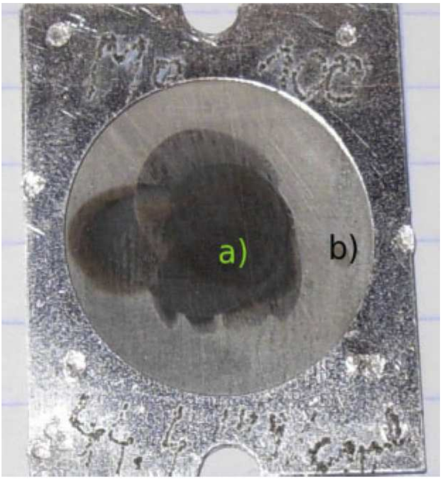
First measurement of diagonal E2 matrix elements using Coulex of radioactive beam

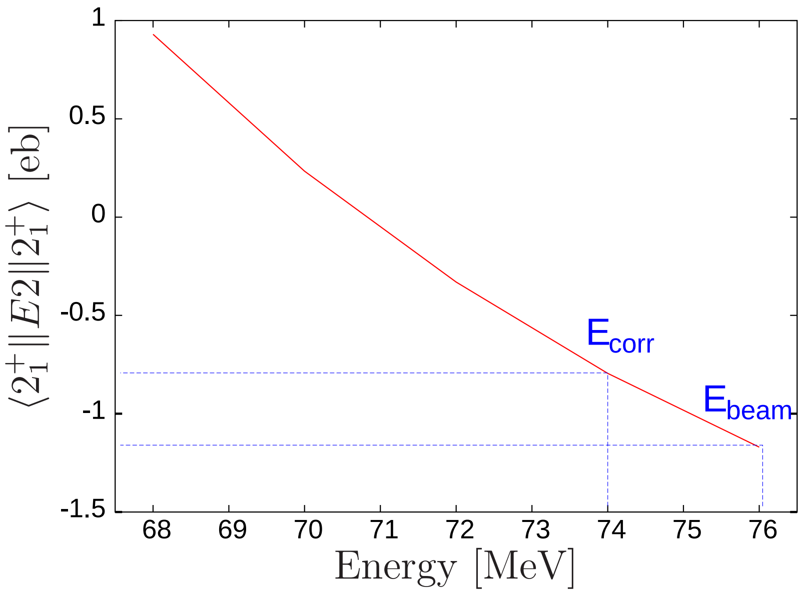
E. Clément *et al.* Phys. Rev. C75, 054313 (2007)

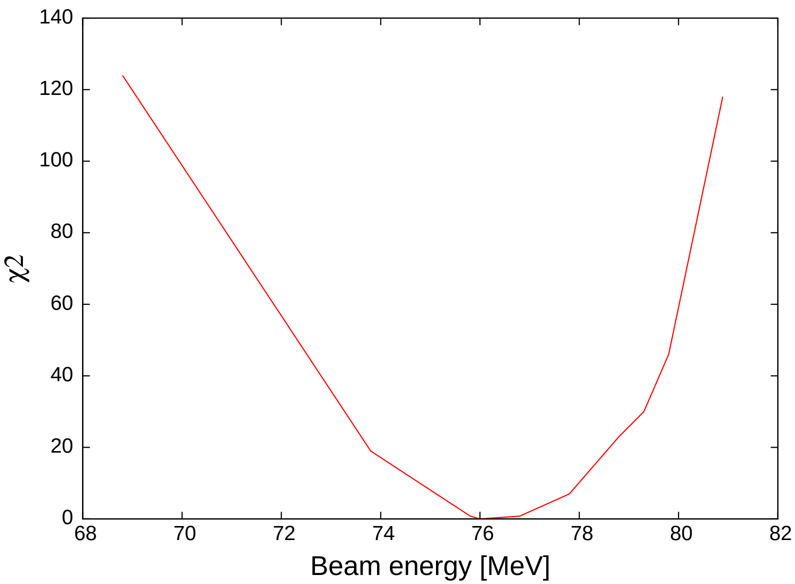
Incident energy

- Strong dependence of multi-step excitation and reorientation effect on beam energy
- Correct beam energy required!





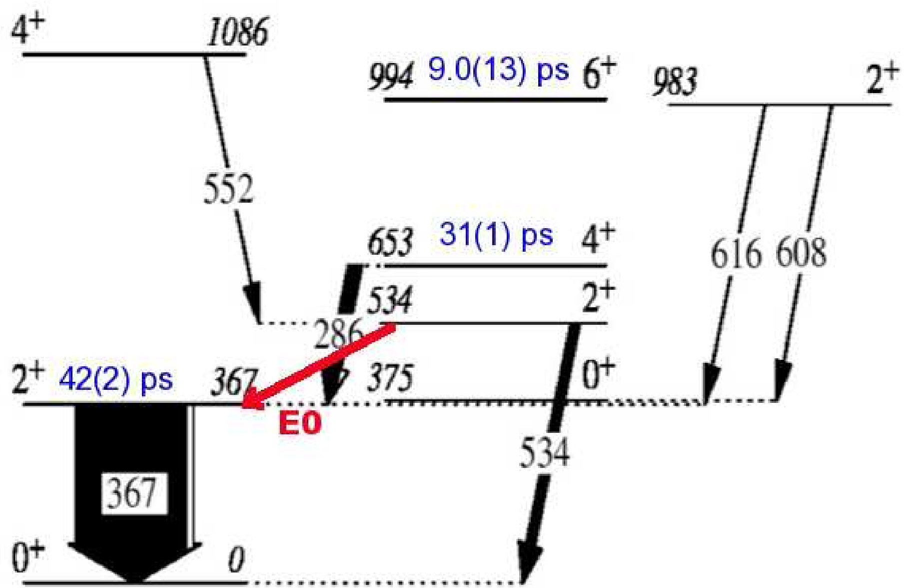




K. Wrzosek-Lipska, PhD thesis (2011)

E0 strengths

- decay branch unvisible for Ge detectors
- important for 0^+ states (^{74}Kr, ^{100}Mo,...) and heavy nuclei



α $(2_2^+ \rightarrow 2_1^+)$ in ^{184}Hg: 23(5)

E. Rapisarda *et al.*, to be published

- electron spectroscopy measurements for strongly converted transitions?