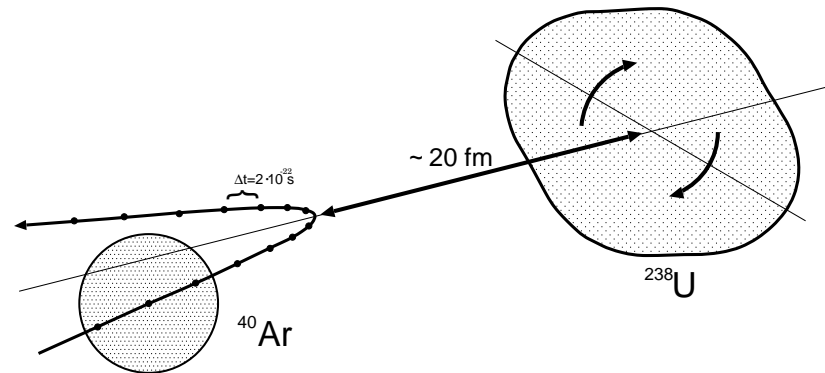

Theoretical description of low-energy Coulomb excitation

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- Why Coulomb excitation?
- Basic introduction to Coulex theory
- Approximations and limitations

Coulomb excitation: what's so great about it?

- population of excited states via purely electromagnetic interaction between the collision partners



- renaissance of the technique as ideally suited for state-of-the-art RIB facilities:
 - beam energies available perfect for Coulomb excitation (2-5 MeV/A)
 - high cross sections (excitation of 2_1^+ : barns)
 - practical at the neutron-rich side
 - easy way to access non-yrast states and study their properties

Basic facts about Coulex

- Due to the purely electromagnetic interaction the nucleus undergoes a transition from state $|i\rangle$ to $|f\rangle$.
- Then it decays to the lower state, emitting a γ -ray (or a conversion electron).
- The matrix elements $\langle f||M(E2)||i\rangle$ describe the excitation and decay pattern \rightarrow they are directly connected with γ -ray intensities observed in the experiment.
- In the intrinsic frame of the nucleus they are related to the deformation parameters.

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Safe energy

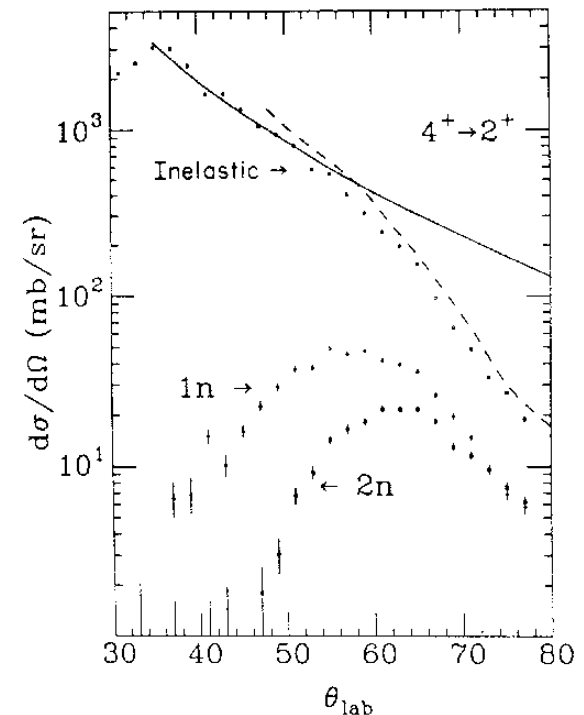
- **Cline's "safe energy" criterion:** purely electromagnetic interaction if the distance between nuclear surfaces is greater than 5 fm

$$D_{min} = 1.25 \cdot (A_p^{1/3} + A_t^{1/3}) + 5.0 \quad [\text{fm}]$$

- empirical criterion based on systematic studies of inelastic and transfer cross-sections at beam energies of few MeV/A

W.J. Kernan et al. / Transfer reactions

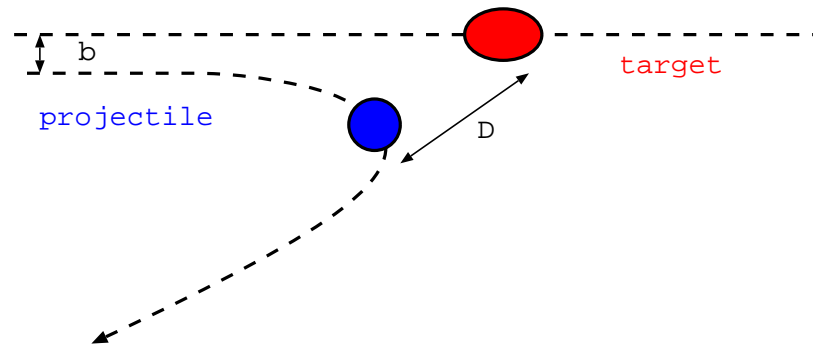
$^{164,163,162}\text{Dy}(^{116}\text{Sn}, ^{118,117,116}\text{Sn})^{162}\text{Dy}$



- other criteria established for high-energy Coulex
- one-neutron sub-barrier transfer recently observed in Coulex of ^{42}Ca on ^{208}Pb

"Safe energy" requirement

...is due to the fact that D_{min} has to be sufficiently large.
But it's not the only way to ensure it!



$$E(\theta_{CM}) = 0.72 \frac{Z_P Z_T}{D_{min}} \cdot \frac{A_P + A_T}{A_T} \left(1 + \frac{1}{\sin\left(\frac{\theta_{CM}}{2}\right)} \right) \quad [MeV]$$

Two possibilities to prepare an experiment:

- choose adequate beam energy ($D > D_{min}$ for all θ)
low-energy Coulomb excitation
- limit scattering angle, i.e. select impact parameter $b(E_b, \theta) > D_{min}$
high-energy Coulomb excitation

Semi-classical approximation vs full quantum treatment

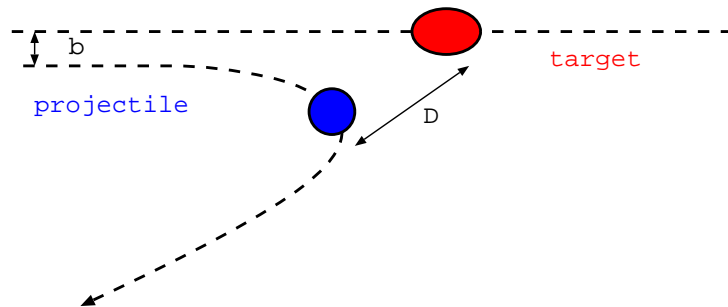
- Electromagnetic interaction well-known → one can easily calculate Coulomb excitation cross section for any states of the investigated nucleus when its internal structure is known (i.e. matrix elements of electromagnetic transitions)
 - Straightforward method – quantum mechanical treatment: high number of partial waves, coupled channel equations... **not very practical :(**
 - Simplified and replaced by a **semiclassical approach** without any significant loss of accuracy

Simplified description of Coulomb excitation

- Projectile is moving along a hyperbolic orbit and excitation of nuclear states is caused by the time-dependent electromagnetic field between the collision partners
- The trajectories are described by the classical equations of motion, quantum mechanics is used to describe the effect of the electromagnetic field on the nucleus
- Other simplifying assumptions:
 - small energy transfer
 - interaction expanded in a multipole series and only monopole-multipole terms taken into account
 - time separation of the collision (10^{-19} s) and deexcitation process ($>10^{-13}$ s)

Validity of classical trajectories

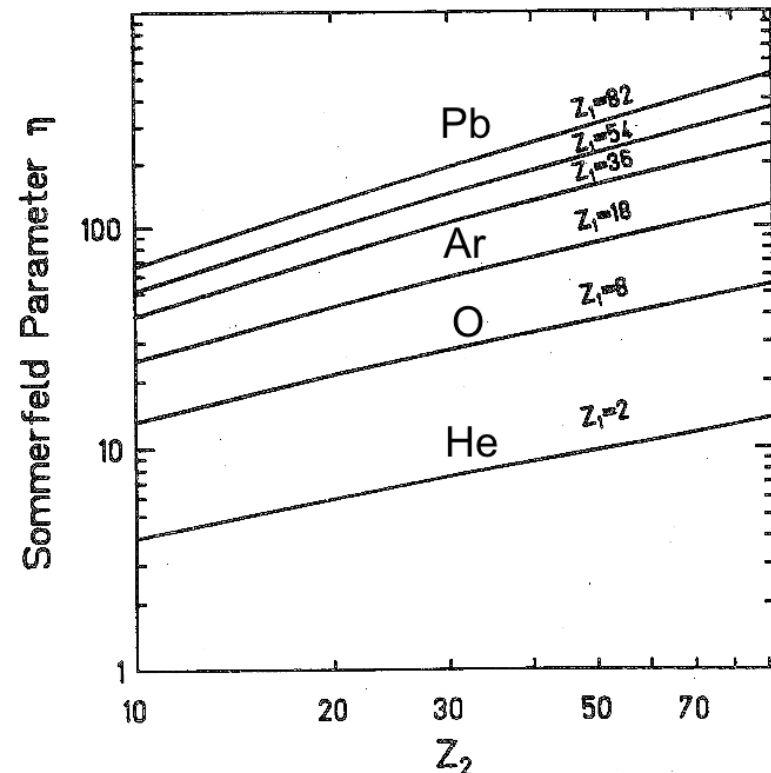
- trajectories can be described by the classical equations of motion, excitation process is described using quantum mechanics.



$\lambda_{\text{projectile}} \ll D$
 \Rightarrow Sommerfeld parameter η

$$\eta = \frac{D}{2\lambda} = \frac{Z_p Z_t e^2}{\hbar v} \gg 1$$

- condition well fulfilled in heavy-ion induced Coulomb excitation
- semiclassical treatment is expected to deviate from the exact calculation by terms of the order $\sim 1/\eta$: most important systematic error in Coulex analysis



Limitation to monopole-multipole terms

The excitation process can be described by the time-dependent Hamiltonian H :

$$H = H_P + H_T + V(r(t))$$

$H_{P/T}$: free Hamiltonian of the projectile/target nucleus

$V(t)$: time-dependent electromagnetic interaction

If the wave function is expressed by eigenfunctions of the free $H_{P/T}$:

$$\psi(t) = \sum_n a_n(t) \phi_n$$

one gets a set of coupled equations for time-dependent excitation amplitudes $a_n(t)$

$$i\hbar \frac{da_n(t)}{dt} = \sum_m \langle \phi_n | V(t) | \phi_m \rangle \exp(i(E_n - E_m)/\hbar) a_m(t)$$

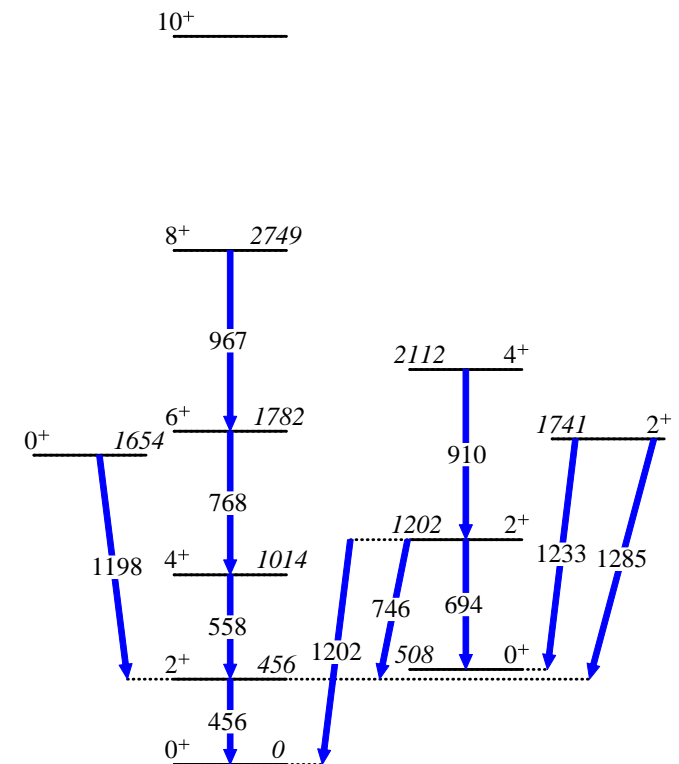
Then $V(t)$ can be expanded in multipoles:

$$\begin{aligned} V(r(t)) &= Z_T Z_P e^2 / r && \text{monopole-monopole (Rutherford) term} \\ &+ \sum_{\lambda\mu} V_P(E\lambda, \mu) + \sum_{\lambda\mu} V_T(E\lambda, \mu) && \text{electric multipole-monopole excitation,} \\ &+ \sum_{\lambda\mu} V_P(M\lambda, \mu) + \sum_{\lambda\mu} V_T(M\lambda, \mu) && \text{magnetic excitation (small at low } v/c) \\ &+ \text{higher order multipole-multipole terms (neglected – estimated at } \sim 0.5 \% \end{aligned}$$

Multi-step excitation and coupled equations

$$i\hbar \frac{da_n(t)}{dt} = \sum_{m,\lambda} \langle \phi_n || M\lambda || \phi_m \rangle \exp(i(E_n - E_m)/\hbar) a_m(t)$$

- in heavy-ion induced Coulomb excitation the interaction strength gives rise to multiple excitation
- a nuclear state can be populated directly, via several intermediate states
- excitation probability of an individual state may depend on many matrix elements involved in different excitation paths
- high number of coupled equations for excitation amplitudes $a_n(t)$
- dedicated data analysis codes needed



Deexcitation process

For a given set of matrix elements $\langle \phi_n \| M\lambda \| \phi_m \rangle$ the set of coupled equations

$$i\hbar \frac{da_n(t)}{dt} = \sum_{m,\lambda} \langle \phi_n \| M\lambda \| \phi_m \rangle \exp(i(E_n - E_m)/\hbar) a_m(t)$$

is solved in order to determine level populations.

The same set of matrix elements describes the deexcitation process:

$$P(M\lambda; I_i \rightarrow I_f) = \frac{8\pi(\lambda + 1)}{\lambda((2\lambda + 1)!!)^2} \cdot \frac{1}{\hbar} \cdot \left(\frac{E_\gamma}{\hbar c} \right)^{2\lambda+1} \cdot B(M\lambda; I_i \rightarrow I_f)$$

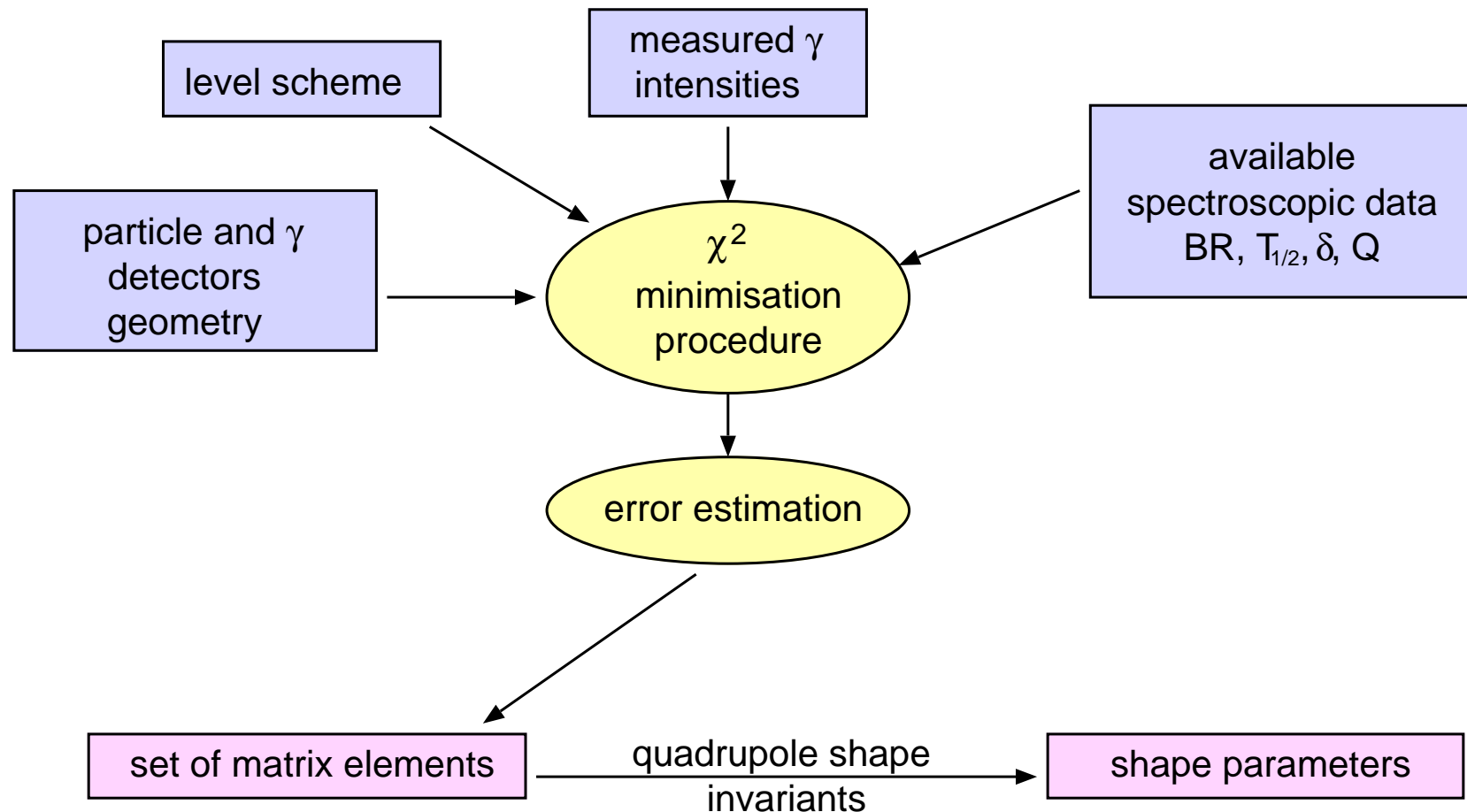
$$B(M\lambda; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} \langle I_f \| M\lambda \| I_i \rangle^2$$

The calculation includes effects influencing γ -ray intensities: internal conversion, γ -ray angular distribution, its attenuation due to finite size of Ge detectors, deorientation

GOSIA code

GOSIA: Rochester - Warsaw semiclassical Coulomb excitation
least-squares search code

Developed in early eighties by T. Czosnyka, D. Cline, C.Y. Wu (Bull. Am. Phys. Soc. 28 (1983) 745.) and continuously upgraded



Approximations used in GOSIA

1. semi-classical approximation
 - symmetrisation of the trajectory to take into account the energy transfer
2. limitation to the monopole-multipole term
3. other effects taken into account in the description of the excitation process:
 - correction for the dipole polarisation effect: quadrupole interaction $V(E2)$ multiplied by a factor

$$1 - d \cdot \frac{E_p A_t}{Z_t^2 (1 + A_p/A_t)} \frac{a}{r}$$

where $d = 0.005$ (empirical E1 polarisation strength, from photo-nuclear absorption cross section or GDR energy + dipole sum rule)

Alder and Winther, Coulomb excitation, appendix J

important for high-lying levels, high CM angles, heavy beams: ^{104}Ru - 10% change of population of 10_{γ}^{+} if effect increased 2 times

- integration over scattering angles covered by particle detectors and incident energy (beam stopping in the target) - changing meshpoints may give an effect of few %, especially for multi-step excitation

Effects taken into account when describing decay

- start from statistical tensors calculated in the excitation stage
 - information on excitation probability and initial sub-state population
 - cascade feeding from higher-lying states
 - deorientation of the angular distribution (due to recoil in vacuum):
Brenn and Spehl two-state model:
 ^{104}Ru - 2% change of matrix elements if effect increased by 20%
 - relativistic transformation of solid angles
 - attenuation due to finite size of gamma-ray detectors
 - simplified (cylindrical) detector geometry
-
- all approximations have usually an effect $\sim 5\%$ on gamma-ray intensities (often similar to statistical uncertainties, increasing with number of steps needed)
 - uncertainties lower than this are rather suspicious (unless they reflect the precision of a lifetime measurement, but the quality of such measurement should also be verified)

Number of parameters versus number of data points

- number of matrix elements coupling low-lying states is higher than number of transitions observed in a Coulex experiment
- some of them have much smaller influence on gamma-ray intensities than others
- even if angular dependence of cross sections is measured, often problem remains underdetermined
 - especially if E1, E3 matrix elements are declared, or for odd nuclei – M1
- additional spectroscopic data needed
 - these data are not used to fix any of the parameters, but enter the χ^2 function exactly like gamma-ray intensities
- in rare very undetermined cases theoretical relations between the ME's may be used (which couplings are negligible, similar, etc...)

Global vs local minimum

Standard question: is this a unique solution, or maybe a different combination of matrix elements can reproduce the experimental data equally well?

Genetic Algorithm in GOSIA: JACOB ([see lecture of P. Napiorkowski](#))

GOSIA:

- often trapped in a local minimum
- various starting points have to be carefully checked (combinations of signs and magnitudes)
- only for very simple cases "plug and play"

JACOB:

- scan of the χ^2 surface, "promising" minima localised
- integration procedure repeated for each of them, real solutions identified
- alternative method for error estimation (in development)